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RESEARCH MEMORANDUM

CONSIDERATIONS INVOLVED IN THE DESIGN OF A ROLL-ANGLE COMPUTER FOR A BANK-TO-TURN INTERCEPTOR

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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INTRODUCTION

At the Ames Aeronautical Laboratory some flight and analog computer studies have been made on the final attack phase of automatic interceptions. The ultimate objective of these studies is to define the behavior of various types of automatic control systems as influenced by a wide range of aerodynamic characteristics. This is a continuing program still in its initial stages; however, it is felt that conclusions of general significance can be drawn from these preliminary studies. One point of particular importance is the necessity for including a gain changer or computer in the azimuth loop of any bank-to-turn airplane or missile. This device translates azimuth error signals into appropriate roll commands. The results to date have shown that the characteristics of this component have a predominant effect on the behavior of an automatic system. It is the purpose of this paper to discuss considerations of importance in the design of a suitable roll-angle computer. In this regard, the present paper complements the analytical work reported in reference 1.

NOTATION

A_L normal acceleration

A_{L_D} desired normal acceleration

A_j, A_k acceleration components proportional to e_j and e_k, respectively

M Mach number

e_j, e_k azimuth and elevation error signals, respectively, with respect to airplane coordinates

g	acceleration due to gravity
h_p	pressure altitude, ft
ϵ_φ	bank angle error, deg
φ	bank angle, deg
η	target angle (fig. 8), deg

DISCUSSION

To point out the range of aerodynamic and inertia characteristics covered by this investigation, figure 1 shows plan views of the three airplanes considered, the SB2C, F-86D, and F-102. These may be considered as representative of a subsonic, transonic, and supersonic interceptor.

A concurrent flight and simulator study has been completed on the SB2C airplane and the results are presented in reference 2. To check the generality of these results, preliminary simulation studies have been conducted on the F-86D and F-102 airplanes, each with a representative automatic control system.

To point out the functions of the various components, figure 2 illustrates, in generalized form, a block diagram that is typical of most present and proposed automatic interceptor systems. Target position and rate of change of position are sensed by self-tracking radar. This information is supplied to a steering-angle computer which calculates the desired lead angle, usually for a lead-pursuit or a lead-collision course, and applies the necessary ballistics corrections. The outputs of the computer are the elevation and azimuth steering errors referred to interceptor coordinates. The elevation steering error commands a normal acceleration (or sometimes a pitch rate) which must be limited by structural and aerodynamic considerations. In the azimuth channel the roll-angle computer calculates a bank-angle error, which in turn commands a roll rate. The rudder servo (not shown in the diagram) generally functions as a yaw damper to hold sideslip angles within acceptable limits. The systems discussed herein are of this general type with some modifications.

SB2C FLIGHT AND SIMULATOR STUDIES

In the SB2C airplane the self-tracking radar was replaced by an optical sighting device and the steering-angle computer was neglected; thus, the line-of-sight angles from the simulated radar were used

directly as steering signals so that the target airplane was tracked only in pure pursuit. In the initial tests of this system, the simplest possible form of roll command was used, as illustrated in figure 3, where the roll-angle command is a linear function of azimuth error. The slope of the line, of course, indicates the gain of the system and it can be seen that the command would increase indefinitely with azimuth error except for eventual saturation of the physical components involved. In the SB2C system the saturation point was well beyond the range of azimuth errors considered in flight.

The behavior of the SB2C with this type of roll computer is shown in figure 4. Plotted here are the responses to initial step lock-on errors in elevation and in azimuth. For comparison both simulated and flight results are shown. It can be seen that the response in elevation shown on the left is reasonably fast with little overshoot. The azimuth response, however, shows a rather large overshoot in addition to a long period sustained oscillation. It was found that no great improvement could be obtained by adjusting system parameters. Increasing gains merely caused a greater overshoot with little or no change in response time. A reduction in gain, on the other hand, further impaired the ability of the airplane to roll in order to correct small errors. It was also found that, as the magnitude of the command input increased, the response became more oscillatory, approaching a condition of roll instability.

Because of the close correlation between flight and simulated results, it was felt that the simulator could be used with confidence in specifying a nonlinear type of roll control which would improve the azimuth response. The roll-angle computer developed on this basis is shown in figure 5. This nonlinear type of control provides high gain for rapid and precise correction of small errors, but still has a low enough gain for large errors to insure stability. It will be noted that for errors greater than about 1° the slope is the same as for the linear command system previously mentioned. Figure 6 shows the great improvement in response that was obtained with the modified roll-angle computer, and it will be noted that the simulator accurately predicted the benefits that were realized in flight.

It should be pointed out that, while this is certainly not an optimum roll-control system, it gave satisfactory results over the limited range of flight conditions for which the airplane was tested. A large number of successful tracking flights were made, including beam attacks as well as tail chases of both maneuvering and nonmaneuvering targets. It was also found that satisfactory responses could be obtained to a variety of initial lock-on situations. Even in the severe case where the target is initially below the flight path of the interceptor the response of the system was very stable.

F-86D SIMULATION STUDIES

To test the generality of conclusions drawn from the SB2C flights, simulator studies were conducted on representative automatic control systems in the F-86D and F-102. In each of these two studies the self-tracking radar and steering-angle computer were represented by a simple time lag and only responses to initial lock-on errors were considered. The control system studied in the F-86D was essentially that developed by the Hughes Aircraft Company and which has been successfully flight tested by them. This system contains a roll-angle error computer, considerably more detailed than that tested in the SB2C, and as a result is more effective over a broader range of flight conditions. When this computer was replaced on the simulator by the simple linear command, figure 7 shows that the system suffered from the same deficiencies as previously noted in the SB2C tests. As the input magnitude is increased there is a definite tendency toward instability which is most readily apparent in the roll-angle response. With the Hughes type of computer, however, there is no unstable tendency in roll; the response shown on the right is typical for inputs of any magnitude, and in no case did the bank angle exceed approximately 60°. (In this study the normal acceleration was limited to 2g.)

DEVELOPMENT OF ROLL-ANGLE COMPUTER

To further examine the concept of the roll-angle computer, a similar automatic system utilizing the F-102 airframe was studied on the simulator and the results closely verified those discussed previously. However, in this case a more explicit approach was used to specify the most desirable characteristics of the roll-angle computer, rather than the cut-and-try method used with the SB2C. The results of these studies point out an inherent limitation of any automatic control system in which the interceptor (or guided missile) must bank to turn, and the problem is encountered even in the absence of nonlinear components such as the rate-limited servo discussed in reference 3.

The correct roll command should not depend on the magnitude of the azimuth error alone but should be a function of target direction as well. For example, if the elevation error is zero the bank angle should never exceed 90° regardless of the magnitude of the azimuth error. The achievement of a proper command for all conceivable target situations requires a particular network which will compute the correct bank-angle error on which to base a roll-rate command. To point out the geometric considerations involved in specifying the most desirable type of computer, figure 8 has been prepared. Here the correct bank-angle error is expressed as a function of relative target position and interceptor roll attitude. The

sketch on the left is a projection of the steering-angle errors in a plane containing the target and normal to the flight path of the interceptor. With the interceptor at a bank angle φ these angles are e_j and e_k. For convenience, the direction of the target in this plane is defined from the vertical by the angle η.

In the sketch at the right an acceleration diagram has been superimposed to show the acceleration commands A_j and A_k which are proportional to the error angles e_j and e_k. When considering the gravity force g, the interceptor must roll through an angle ϵ_φ and attain a normal acceleration A_{L_D} in order to produce a resultant acceleration in the direction of the target. The angle ϵ_φ is thus the instantaneous bank-angle error and may be expressed as a trigonometric function of the variables A_j, A_k, and φ. The arc tangent function shown at the bottom of figure 8

$$\epsilon_\varphi = \tan^{-1} \frac{A_j - g \sin \varphi}{A_k + g \cos \varphi}$$

provides an exact calculation for error angles as large as 180°. An alternate expression is the arc sine function

$$\epsilon_\varphi = \sin^{-1} \frac{A_j - g \sin \varphi}{A_{L_D}}$$

where

$$A_{L_D} = \sqrt{(A_j - g \sin \varphi)^2 + (A_k + g \cos \varphi)^2}$$

Because of the first and second quadrant ambiguity in the arc sine function, however, this expression will never indicate a bank-angle error greater than 90°.

Because of practical difficulties in mechanizing inverse trigonometric functions, it is desirable to find a simpler expression for ϵ_φ. Subsequent simulator studies showed that the most satisfactory simplification was to use a small angle approximation to the arc sine function,

$$\epsilon_\varphi \approx \frac{A_j - g \sin \varphi}{|A_j| + |A_k| + K}$$

The constant K is necessary to prevent the roll command from becoming indeterminate as the error signals approach zero, and in this sense is an approximation of the gravity component. The bank-angle error shown here can then be considered as a roll-rate command.

To show how the roll command varies with relative target position, figure 9 has been prepared. In this case the steering-error signals A_j and A_k are assumed to be very large compared to the gravity terms, so that the true bank-angle error is simply the difference between η and φ. The arc tangent function plots as a straight line giving an exact solution of ϵ_φ, and in the most extreme cases indicates a bank-angle error of 180°. This is the case where the target is directly below the flight path of the interceptor. With the arc sine function, however, the computed bank-angle error never exceeds 90° for any relative target position, and actually returns to 0° as the true error approaches 180°. The small angle approximation indicates a somewhat smaller value throughout the entire range, but by applying a multiplying factor the level may be adjusted to any desired value. For example, with a factor of $\pi/2$ the two curves are almost identical from 0° to 180°.

Since the interceptor roll rate is proportional to bank-angle error, it can be seen that with an arc tangent calculation, maximum roll rates will be experienced for target angles near 180°. In most practical cases, however, where this negative elevation error is not extremely large, it is more desirable for the interceptor to pitch down without rolling. The arc sine function permits this type of maneuver by restricting the roll rate for target angles near 180°. Another point of interest is the variation in computed bank-angle error with azimuth steering error for a particular target angle. Figure 10 shows the results for the particular case in which the initial target angle is 90° (0° elevation error), and the bank angle of the interceptor is 0°. Again, the exact calculation and the arc sine approximation are compared. It can be seen that as the azimuth error becomes large the roll angle approaches a constant value equal to η which in this case is 90°. Similar curves may be plotted for other target angles and indicate that the correct roll-angle command is a function of elevation as well as azimuth error. The shape of the curve indicates the same effective gain variation that was previously noted in the discussion of the SB2C system. Also shown is the apparent bank-angle error obtained with the linear command. Here the bank-angle error is independent of target angle and would increase indefinitely as the azimuth error became large.

F-102 SIMULATION

In the simulation studies of the F-102 system all three types of roll computers shown here were included. Figure 11 is a comparison of responses with the linear command and with the arc sine approximation for an initial azimuth error. As previously illustrated, the response with the linear command tends to become unstable as the size of the input is increased, but with the arc sine computer the response is rapid and stable for commands of any magnitude. For this case of a 90° target angle, the arc tangent computer gave practically identical results

as would be expected from figure 9. However, for situations in which the target angles were greater than 90° there was considerable difference in response. In fact, when η was near 180° the use of the arc tangent function resulted in violent responses to small negative elevation errors with the interceptor rolling well beyond 360°. The situation was improved somewhat by physically limiting the roll command to 90° through this region, thus preventing the very high roll rates. Even with this limiting modification it can be seen in figure 12 that the interceptor still rolled as much as 360° in correcting a -5° elevation error. In other cases in which the initial conditions were slightly different, the interceptor rolled 180° and then reversed its direction of roll and returned to 0°. With the arc sine computer, however, there was very little tendency to roll during this same maneuver. For the less critical case of a 5° azimuth error and a -5° elevation error, the responses do not differ greatly. Roll angles reached about 130° in each case with the arc tangent computer calling for somewhat higher initial roll rates.

CONCLUDING REMARKS

In this paper it has been shown that a gain-changing device or computer is required in the azimuth channel of a bank-to-turn automatic interceptor or guided missile and that the characteristics of this computer have an important bearing on the behavior of the vehicle. One particular type of computer which provides roll-rate commands proportional to an approximation of the bank-angle error was found to have the following desirable qualities: It is fairly simple to mechanize; it provides effective roll-rate limiting; it prevents violent rolling during pitch-down maneuvers; and finally, by approximating the effect of gravity, it eliminates roll uncertainty when the steering errors approach zero.

Although not evident from the results shown, roll-rate limiting has the beneficial effect of minimizing inertial coupling between the pitch and yaw modes. In all of the airplanes studied here, these cross-coupling effects appeared to be negligibly small. In the initial simulator studies, five degrees of freedom were assumed and all the cross-coupling terms were included. But when these terms were deleted from the equations of motion, there was very little difference in the responses to initial step commands. In addition to low roll rates which never exceeded 120° per second, this fact was partly due to low sideslip angles resulting from either high directional stability or suitable yaw damping, and partly due to the effective filtering action of the electronic and kinematic feedbacks in the automatic system.

As pointed out earlier, the simulation and flight results discussed in this paper were based on responses to step lock-on commands. Factors such as radar noise, variation in range during an actual attack, and target maneuvers, will undoubtedly influence the final design of any system.

The step command, however, is a severe test and can give considerable insight into the relative merit of different types of systems.

Ames Aeronautical Laboratory
National Advisory Committee for Aeronautics
Moffett Field, Calif., Apr. 18, 1955

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1. Mathews, Charles W.: Study of the Attack of an Automatically Controlled Interceptor on a Maneuvering Bomber With Emphasis on Proper Coordination of Lift-Acceleration and Roll-Angle Commands During Rolling Maneuvers. NACA RM L54E27, 1954.

2. Turner, Howard L., Triplett, William C., and White, John S.: A Flight and Analog Computer Study of Some Stabilization and Command Networks for an Automatically Controlled Interceptor During the Final Attack Phase. NACA RM A54J14, 1955.

3. Schmidt, Stanley F., and Triplett, William C.: Use of Nonlinearities to Compensate for the Effects of a Rate-Limited Servo on the Response of an Automatically Controlled Aircraft. NACA TN 3387, 1955.

PLAN VIEWS OF THE THREE EXAMPLE INTERCEPTORS

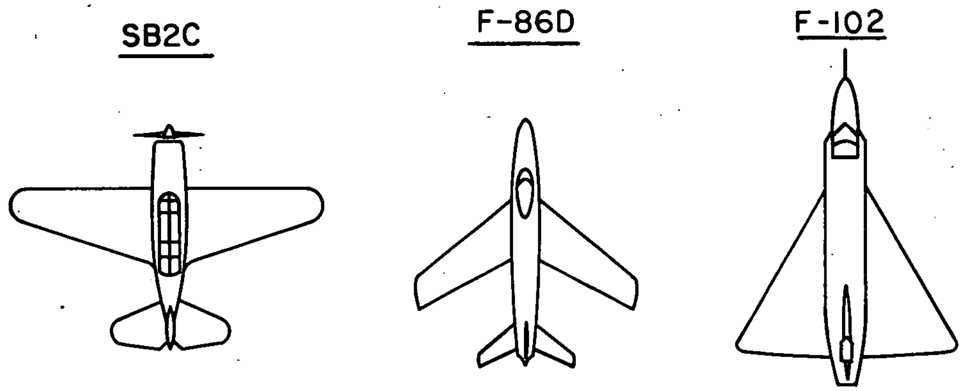


Figure 1.

GENERALIZED BLOCK DIAGRAM OF AUTOMATIC INTERCEPTOR CONTROL SYSTEM

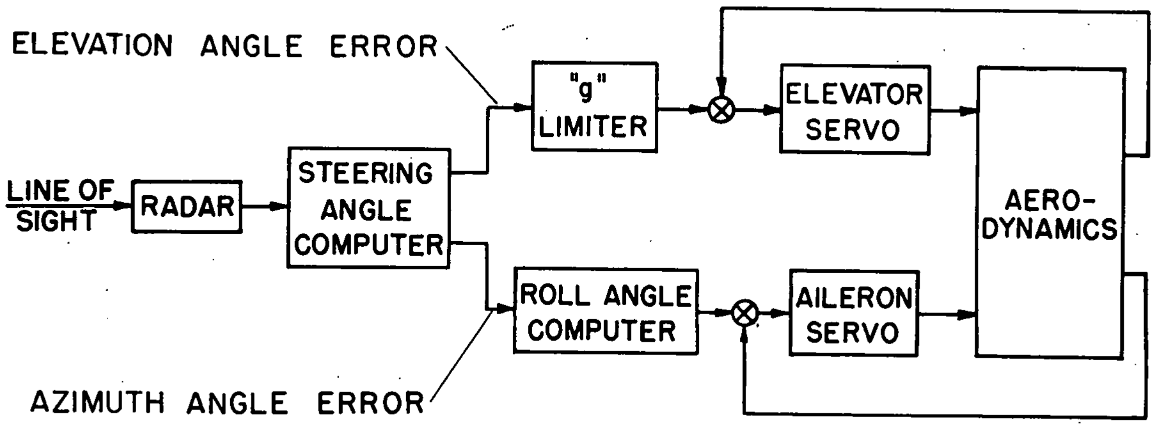


Figure 2.

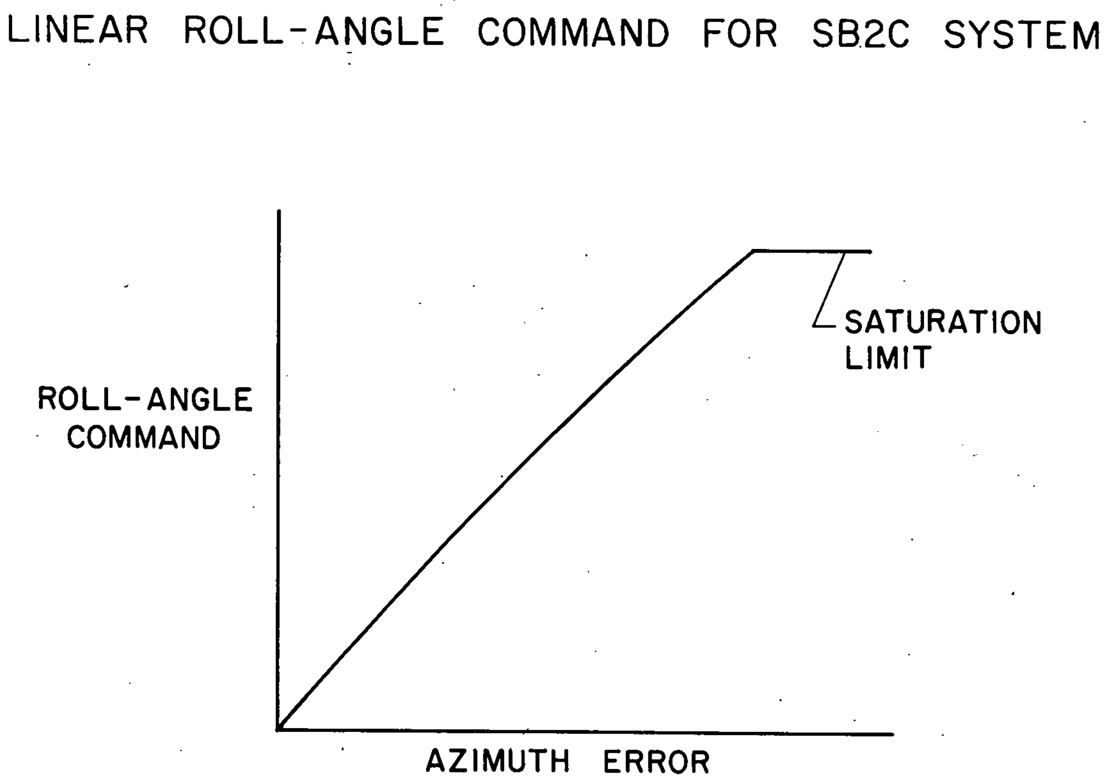


Figure 3.

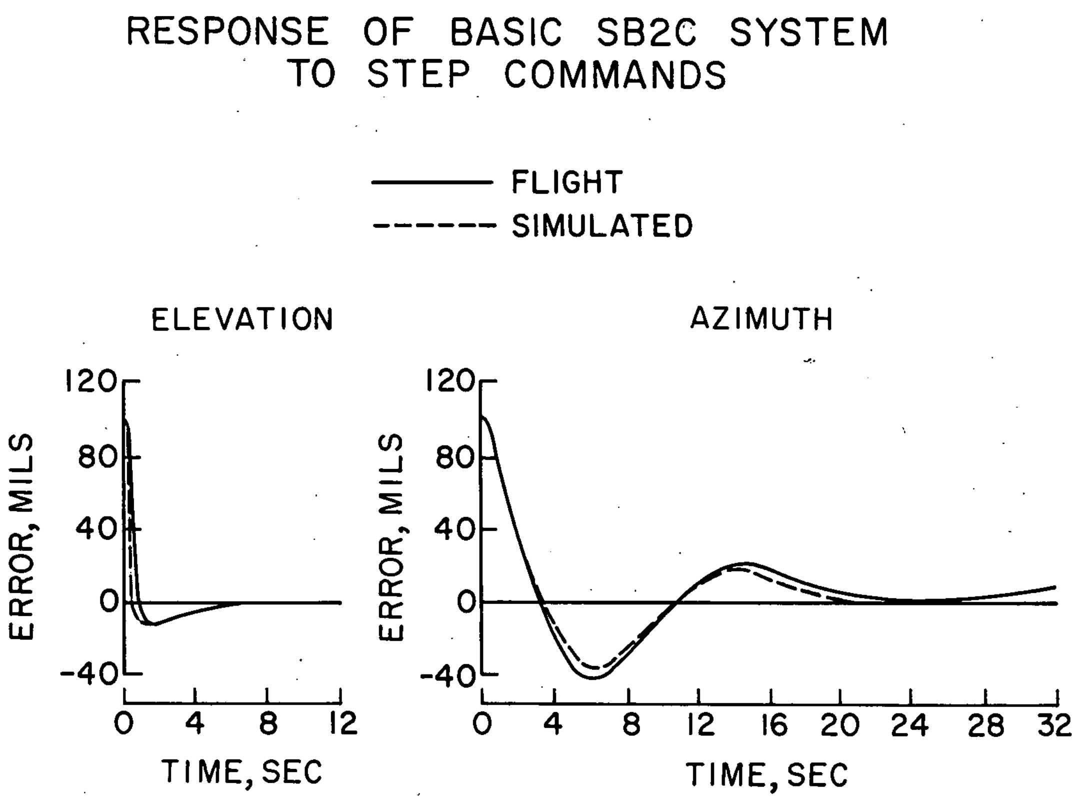


Figure 4.

ROLL-ANGLE COMMANDS USED IN SB2C SYSTEM

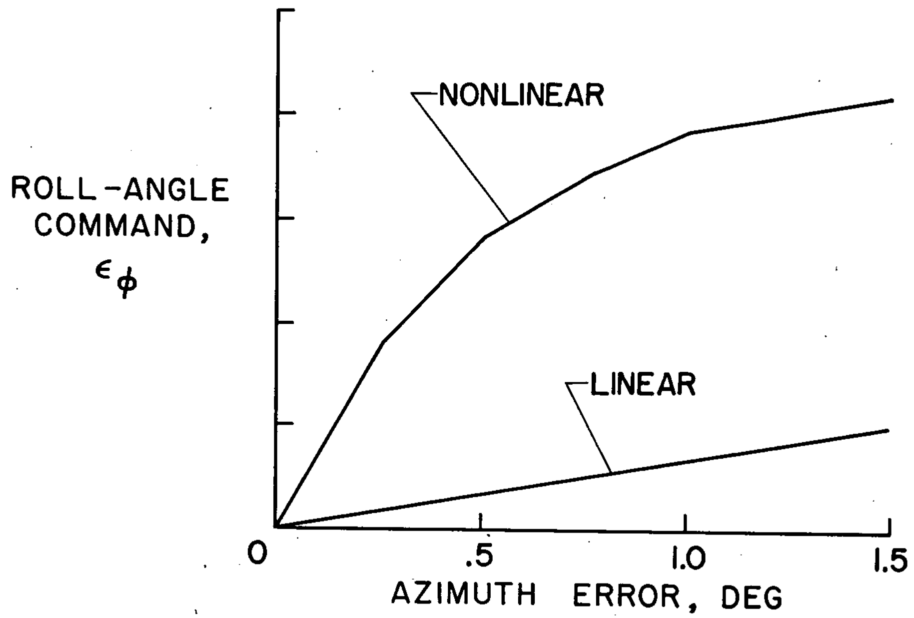


Figure 5.

SB2C RESPONSE TO INITIAL AZIMUTH COMMAND

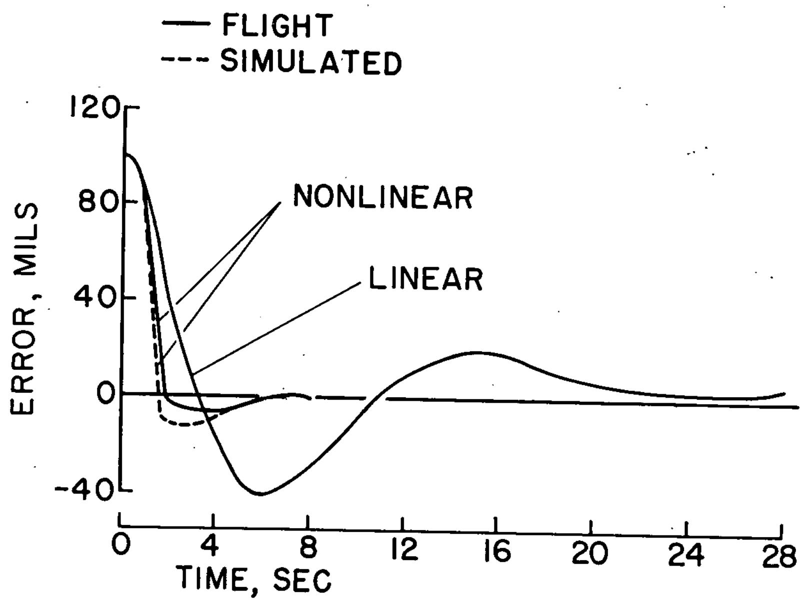


Figure 6.

SIMULATED RESPONSE OF F-86D TO INITIAL AZIMUTH ERRORS $h_p = 40{,}000$ FT, M = .9

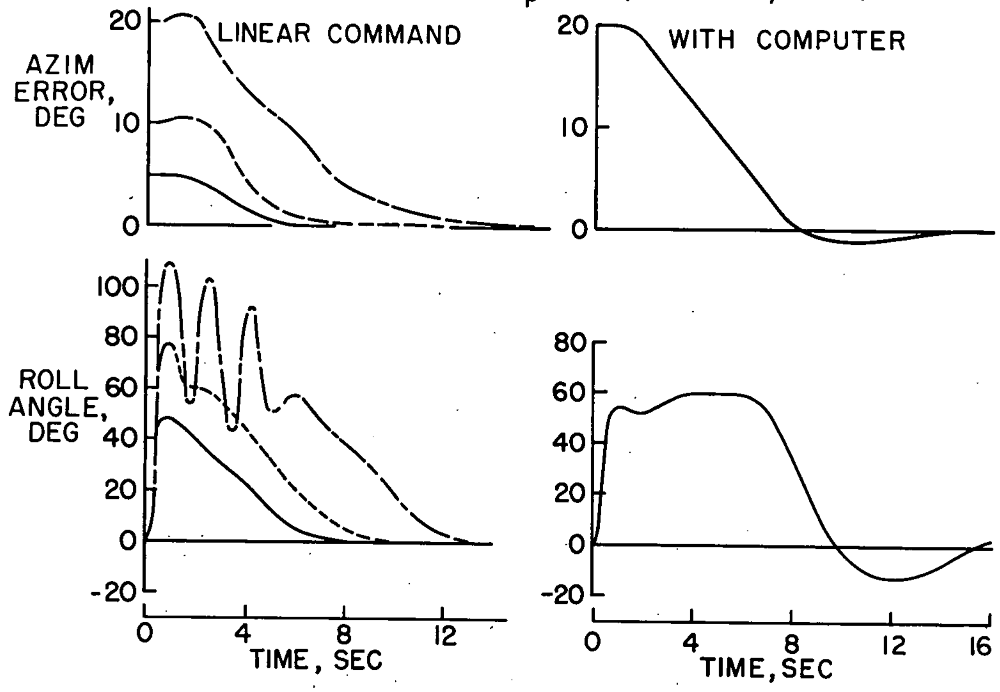
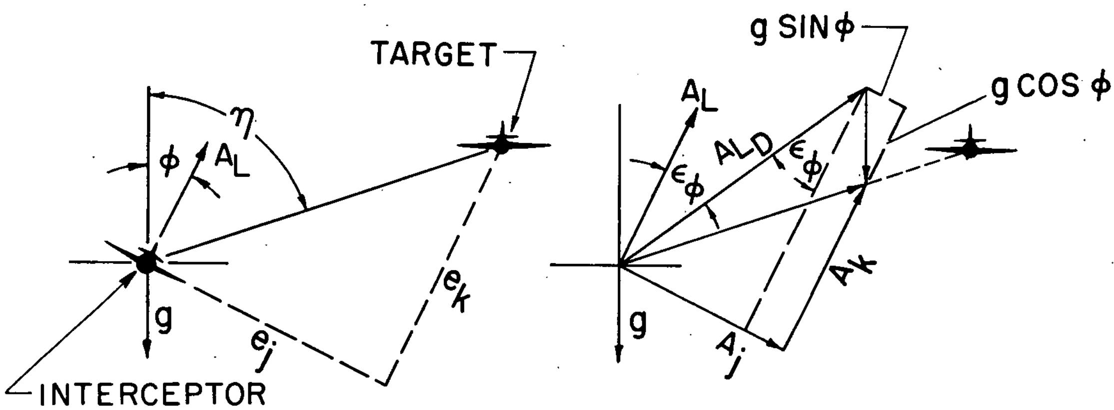


Figure 7.

CALCULATION OF BANK-ANGLE ERROR



(1) $\epsilon_\phi = \text{TAN}^{-1} \dfrac{A_j - g\ \text{SIN}\,\phi}{A_k + g\ \text{COS}\,\phi}$

(2) $\epsilon_\phi = \text{SIN}^{-1} \dfrac{A_j - g\ \text{SIN}\,\phi}{A_{L_D}} \approx \dfrac{A_j - g\ \text{SIN}\,\phi}{|A_j| + |A_k| + K}$

Figure 8.

COMPUTED ROLL-ANGLE ERROR FOR LARGE STEERING ERRORS

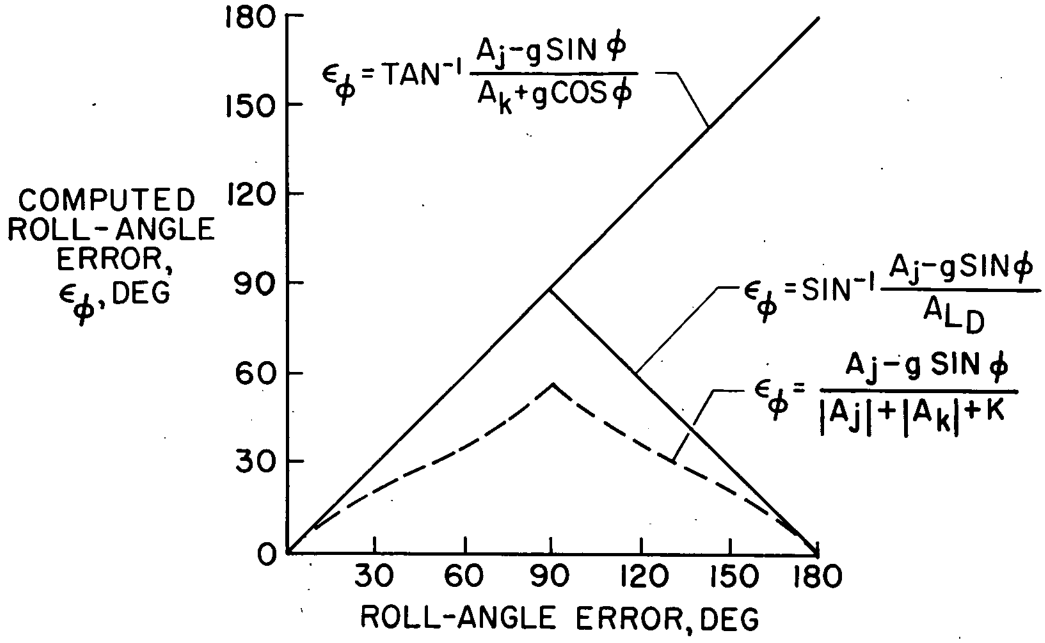


Figure 9.

ROLL-ANGLE ERROR AS A FUNCTION OF AZIMUTH ERROR
η = 90 DEG

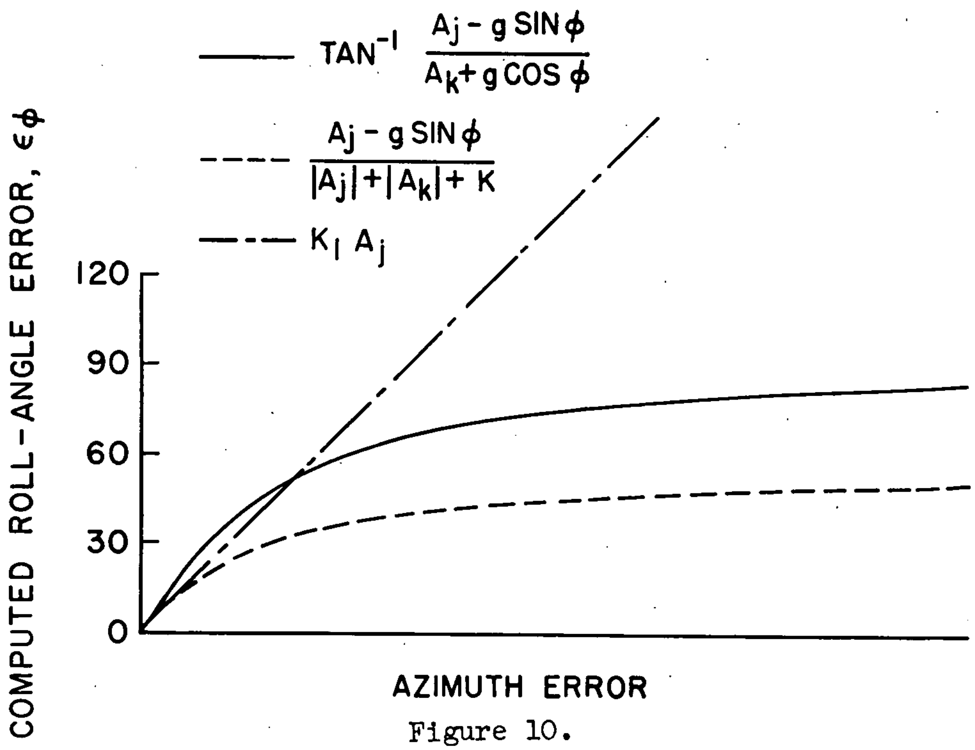


Figure 10.

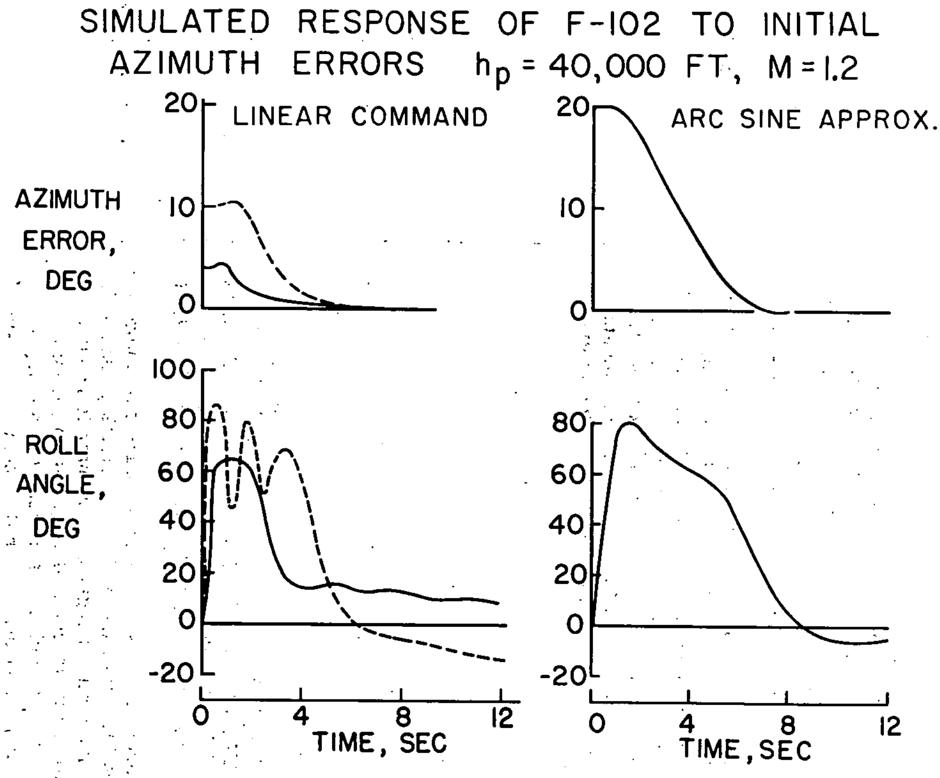


Figure 11.

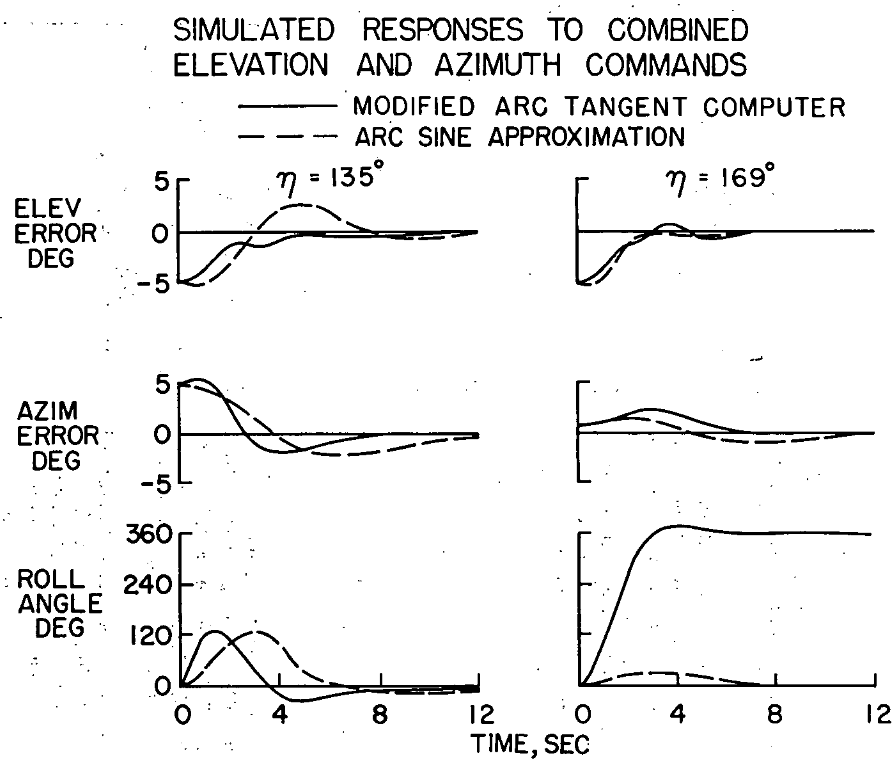


Figure 12.